Indian J. Phys. **73B** (6), 889-898 (1999)

IJP B
— an international journal

Neutron stars and accretion disks in low mass X-Ray binaries

Sudip Bhattacharyya
Indian Institute of Astrophysics
Bangalore,India
E-mail : sudip@iiap.ernet.in

Abstract. In this paper, the evolution of Low Mass X-ray Binaries will be described briefly. Accretion disks around neutron stars in LMXBs will also be discussed and the importance of general relativistic effect of rapid rotation on the disk will be pointed out. Millisecond pulsars and some of the observed aspects of LMXBs will be mentioned.

Keywords : Neutron stars, accretion, X-ray binaries, Pulsars

PACS Nos. : 97.60.Jd, 97.10.Gz, 97.80.Jp, 97.60.G

1. Introduction

We observe x-ray point sources in the sky, which produce very high x-ray luminosity (typically $\sim 10^{37}$ ergs/sec.). Nuclear energy release (efficiency $\sim 0.7\%$) can not explain such a high power. For example, the luminosity (which is mostly optical) of the sun (which is powered by nuclear energy) is 4×10^{33} ergs/sec.. The spectra of the said x-ray sources are also not similar to the stellar spectra. It is believed that this high luminosity is due to gravitational energy release via accretion in a binary stellar system.

If one of the components in a binary system is a compact star (black hole, neutron star or white dwarf) while the other is a normal star, then matter from the latter may flow towards the former, provided certain conditions are fulfiled. Due to initial angular momentum, this matter may form a disk, which is called accretion disk. X-ray comes from this disk, as well as from the surface of the compact star (except black hole, which has no hard surface). Here we will not consider white dwarf as a compact star,because it is not compact enough to produce such a high luminosity and hard x-ray. In this paper, we will focus mainly on neutron star, though some of the treatments will be valid for black hole too.

Two distinctly different types of x-ray binaries are observed. They are called High Mass X-ray Binary (HMXB) and Low Mass X-ray Binary (LMXB). The former contains a massive normal star, while the latter's normal component is a low mass star. We will focus on LMXBs in this paper.

We also observe millisecond (ms) radio pulsars, which are believed to be rapidly rotating neutron stars in LMXBs (or were in LMXBs for some time). They were probably spun up by accretion induced angular momentum transfer.

The structure of this paper is as follows :

In section 2, the main features of x-ray binaries are mentioned. In section 3, a brief outline of the evolution of LMXBs is given. Section 4 deals with accretion disk and the general relativistic effect of rapid rotation on it. In section 5, ms pulsars and some of the observed features of LMXBs are mentioned. We summarize and discuss the results of the foregoing sections in section 6.

2. X-ray binaries

Strong ($L_x \sim 10^{34}$ to 10^{38} ergs/sec.) galactic x-ray sources belong to two categories, as mentioned in section 1. We are giving their properties below. Here we will consider only neutron star x-ray binaries.

2.1. *HMXB*

These sources contain early type normal stars (generally, O or B type). So the optical spectra of HMXBs are dominated by the spectra of their normal components. We observe hard x-ray spectra ($kT \gtrsim 15$ keV in exponential fit [5]) and $\frac{L_{opt}}{L_x} > 1$ for these systems. they show regular x-ray pulsations, but no x-ray bursts (discussed later), which indicates that their surface dipole field strengths are typically of the order of 10^{11} to 10^{13} Gauss. HMXBs are found to be concentrated in the galactic plane. Hence they form a young stellar population (age $< 10^7$ yr.), which is consistent with the fact that they contain early type normal stars.

As they contain very bright normal components,which can be easily detected, one can determine their orbital period, observing the regular x-ray eclipses, which are very frequent in these systems. With this and the doppler radial velocity curves of the neutron star (or pulsar) and its companion, and the light curve, the mass of each component, as well as the radius of the normal companion can be determined [12, 13].

2.2. *LMXB*

These systems contain late type normal stars, which can not be detected easily (in fact, normal companions have been detected for very few cases). So the determination of the masses of the components is not possible for most of the cases. We observe softer x-ray spectra ($kT \lesssim 10$ keV in exponential fit

[16]) and $\frac{L_{opt}}{L_x} < 0.1$ for them. LMXBs show x-ray bursts for many cases, but regular x-ray pulsations for very few cases. They concentrate in the galactic centre and globular clusters. They are old systems (age $\sim$ (5 to 15) $\times 10^9$ yr.).

3. Evolution of LMXBs

In a coordinate system co-rotating with the binary, there is pear-shaped equipotential (of the combination of gravitational and centrifugal forces) surface around each component. Going outwards from the two mass centres, at a certain value of potential, these two surfaces touch each other at the first Lagrangian point L_1, located on the connecting line of the centres of two components. This critical equipotential surface through L_1 is called *Roche lobe*. When the secondary (i.e., normal star) fills its Roche lobe, matter from it flows towards the primary (neutron star in our case) through L_1 due to unbalanced pressure (as at L_1, there is a potential maximum along the line connecting two mass centres). In the Roche lobe of the primary, forming an accretion disk, this matter finally falls on the surface of the neutron stars and emits x-ray. This type of mass transfer is called *Roche lobe overflow*. As the wind from a low-mass star is not strong, we neglect the accretion from stellar wind and take Roche lobe overflow as the only mechanism for mass transfer.

The following formulae will be used in the calculation of the evolution of LMXBs.

$$4\pi^2 a^3 = G(M_1 + M_2)M_\odot P^2 \qquad \text{(Kepler's law)}. \tag{1}$$

$$R_{2L} = 0.462 \left(\frac{M_2}{M}\right)^{\frac{1}{3}} \qquad \left(\text{For } \frac{M_2}{M_1} \lesssim 0.8, \text{ see [6]}\right). \tag{2}$$

$$b_1 = a\left(0.5 - 0.227 \cdot \log\left(\frac{M_2}{M_1}\right)\right) \qquad \text{(see [4])}. \tag{3}$$

$$\frac{M_2}{M_\odot} = \frac{R_2}{R_\odot} \qquad \text{(For low mass stars)}. \tag{4}$$

In the above formulae, M_1 and M_2 are the masses (in $M_\odot$) of the primary and the secondary respectively, M (in $M_\odot$) is the total mass of the system, R_2 is the actual radius of the secondary, R_{2L} is the radius of a sphere of the same size as the Roche lobe of the secondary, a is the binary separation , P is the orbital period and b_1 is the separation between the primary centre and L_1.

From the above formulae and the mass-radius relations of different types of stars, it can be shown that, there may be ***degenerate stars*** and ***helium main sequence stars*** with the orbital period less than 80 minutes; ***main sequence*** (i.e., hydrogen rich unevolved solar-type) ***stars*** in the orbital period range 80 minutes to 10 hours; and evolved, i.e., *(sub-)giant stars* with the orbital period greater than 10 hours in the LMXBs as the normal components [2]. Here we will focus on main sequence secondary.

A main sequence star can fill its Roche lobe, only if the Roche lobe shrinks to the size of the star. This is achieved when the system loses considerable amount of angular momentum and hence the separation between the two components decreases. Two mechanisms are known for angular momentum loss : gravitational radiation and magnetic braking. The former is a general relativistic effect, while the latter is due to the fact that the angular momentum of the secondary (and hence of the whole system) is carried away by the stellar wind (which is tied and hence co-rotating with the secondary (Ferraro's law of isorotation, [10])). Hence the rate of change of angular momentum,

$$\dot{J} = \dot{J}_{GR} + \dot{J}_{MB}, \tag{5}$$

where $\dot{J}_{GR}$ and $\dot{J}_{MB}$ are given by,

$$\frac{\dot{J}_{GR}}{J} = \frac{64}{5}\frac{G^3}{c^5}\frac{M_1 M_2 M}{a^4} \quad \text{(see [8, 17])}, \tag{6}$$

$$\text{and } \dot{J}_{MB} = -0.5 \times 10^{-28} \cdot k^2 f^{-2} M_2 R_2^4 \omega^3 \quad \text{(see [17])}, \tag{7}$$

where, k is the radius of gyration of the secondary, f is a constant (of the order of unity) and ω is the spin angular speed of the secondary.

We assume, that the secondary star restores its thermal (& hence dynamic) equilibrium (which is continuously disturbed by the mass loss), i.e., relation 4 is maintained, for $t_{\dot{M}} > t_{KH}$, where, accretion rate time scale $t_{\dot{M}} = \frac{M_2}{\dot{M}}$ ($\dot{M} \longrightarrow$ accretion rate) and Kelvin-Helmholtz time scale $t_{KH} = \frac{GM_2^2}{R_2 L_2}$ (L_2 is the current luminosity of the secondary). We also assume, that, $R_{2L} = R_2$ is always maintained which may be true in a long term average sense [6]. Now the total angular momentum is
$J = 2.56 \times 10^{-4} \cdot M_1 M_2 M_\odot^2 \cdot \left(\frac{a}{M M_\odot}\right)^{\frac{1}{2}}$ g.m.-c.m.^{2}-sec.$^{-1}$ (a is in c.m.), which gives, using the previous equations,

$$\frac{2\dot{J}}{J} = \frac{\dot{R}_{2L}}{R_{2L}} + \left(\frac{5}{6} - \frac{M_2}{M_1}\right) \cdot \frac{2\dot{M}_2}{M_2}. \tag{8}$$

Using the above equations and assuming $k^2 f^{-2} = 0.076$ [11], we get the differential equation governing the evolution as,

$$\dot{M}_2 = -\frac{1.188 \times 10^{-10} \cdot M^{-\frac{1}{3}}(M - M_2)^2 M_2^{-\frac{2}{3}} + 1.418 \times 10^{-8} \cdot M^{\frac{1}{3}} M_2^{\frac{5}{3}}}{4M - 7M_2}, \tag{9}$$

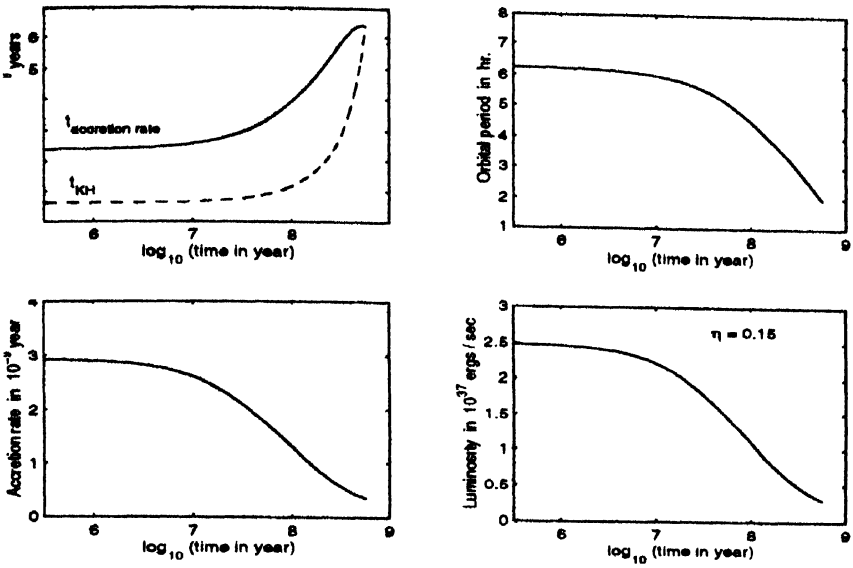

Figure 1. Variation of some relevant quantities of an LMXB with logarithmic time. See text for the details.

where, M_2 is in $M_\odot - yr.^{-1}$ and time is in year.

Solving equation 9 numerically, keeping M constant ($\dot{M}$ is the accretion rate, and not the time variation of M), we get the evolution of M_2 with time and hence the evolution of $\dot{M} = -\dot{M}_2$, a, P_{hr}, L (luminosity of the system), $t_{\dot{M}}$ and t_{KH}, which can be calculated by the previous equations, as well as the equations given below,

$$L = 5.6793 \times 10^{10} \cdot \dot{M}\eta \quad (\text{in } 10^{36} \text{ ergs/sec.}), \tag{10}$$

$$\text{and } \frac{L_2}{L_\odot} = \left(\frac{M_2}{M_\odot}\right)^3 \quad (\text{see [7]}), \tag{11}$$

where η (taken as constant, though it may in general vary (with time) from 0.2 to 0.05) is the efficiency of conversion of accreted rest mass to energy.

We have solved equation 9 for $M = 2.1M_\odot$ and $M_1(t=0) = 1.4M_\odot$ upto a point $t_{\dot{M}} \gtrsim t_{KH}$. Secondary mass reduces from $0.7M_\odot$ to $0.219M_\odot$ (which is sufficient for nuclear burning) in the total time span, which is found to be well below the present age of the universe. The calculated orbital period always remains between 80 minutes and 10 hours, which is expected. Luminosity has been found to be of the order of 10^{37} ergs/sec., which tallies with observational results.

In Figure 1, we have plotted some of these quantities with logarithmic time.

4. Accretion disk

The matter, coming through L_1 towards the neutron star, possesses some initial angular momentum due to the orbital motion of the secondary.This matter will lose some energy by dissipative processes and attain the lowest possible energy for that particular angular momentum; hence is expected to be rotating in a circular Keplerian orbit around the neutron star at a distance R_{circ} from the centre of the primary, given by,

$$R_{circ} v_{\phi}(R_{circ}) = b_1^2 \omega \qquad \text{(see [4])}, \tag{12}$$

where, $v_{\phi}(R_{circ}) = \left(\frac{GM_1 M_{\odot}}{R_{circ}}\right)^{\frac{1}{2}}$ and ω is the binary angular frequency. But dissipative processes will be going on in the ring of matter at R_{circ} , resulting in the loss of energy and redistribution of angular momentum. As the Keplerian angular momentum increases outwards (which is the Rayleigh's criterion [14] for stability), most of the particle will orbit closer and closer to the neutron star losing their angular momentum, while a few particles will go outwards gaining angular momentum. Thus a disk will be formed, as the radial speed of a particle is negligible compared to its transverse speed. We are considering here only geometrically thin, Keplerian disks, though the disk may be thick and non-Keplerian in many real sources.

As LMXB is an old system, we assume that the magnetic field of the neutron star has decayed [2] considerably and the disk touches the surface of the star. For Newtonian case, disk luminosity L_D and surface luminosity L_S (i.e., energy coming from the surface of the neutron star) are equal and hence the irradiation of the disk by the surface layer is significant. Taking the disk as blackbody and time independent, one can calculate its temperature profile $T_{eff}(r)$ (from the flux of energy coming out from the disk) given by,

$$T_{eff}(r) = \left\{\frac{3GM\dot{M}}{8\pi r^3 \sigma} \cdot \left[1 - \left(\frac{R_*}{r}\right)^{\frac{1}{2}}\right]\right\}^{\frac{1}{4}}, \tag{13}$$

where, σ is Stefan's constant, M and R_* are the mass and radius of neutron star respectively, $\dot{M}$ is the accretion rate and r is the radial distance from the centre of the primary. For the calculation of the disk structure and the temperature profile, one may refer [4]. From this sort of temperature profile, one gets multi-coloured blackbody spectrum, which is actually observed from

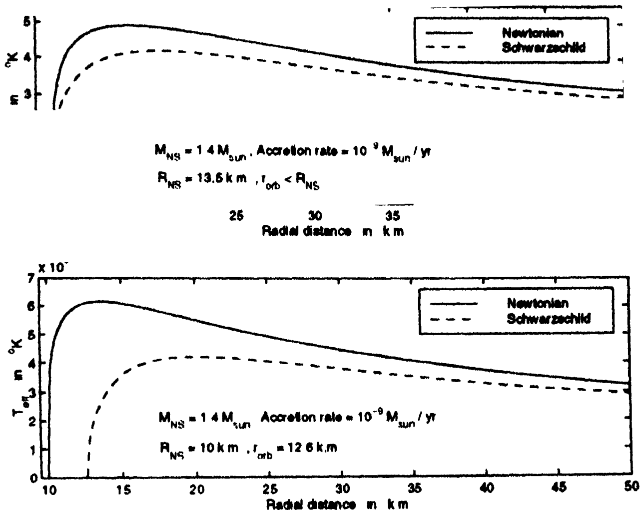

Figure 2 : $T_{eff}(r)$ profiles for different cases. See text for the details.

some neutron star LMXBs (for example, Cygnus x-2). Surface layer gives a single temperature blackbody spectrum.

As neutron star is a very compact object (mass $\sim 1M_{\odot}$ and radius ~ 10 k.m.), general relativistic effect is important inside it and near its surface. General relativity predicts the existence of the innermost stable circular orbit (of radius r_{orb}) [15], inside which there will be no disk and free fall of matter will occur. So the inner radius r_{in} of the disk will be equal to R_* or r_{orb}, whichever is greater. This effect and the calculation in general relativistic (Schwarzschild) way, gives the modified $T_{eff}(r)$ [18] as

$$T_{eff}(r) = \left\{\frac{F(r)}{\sigma}\right\}^{\frac{1}{4}}, \tag{14}$$

where,

$$F(r) = \frac{3m\dot{M}}{8\pi r^{\frac{5}{2}}(r-3m)}\left[\sqrt{r}-\sqrt{r_{in}}+\frac{\sqrt{3m}}{2}\cdot\ln\frac{\cdots-\sqrt{3m})}{(\sqrt{r}-\sqrt{3m})(\sqrt{r_{in}}+\sqrt{3m})}\right], \tag{15}$$

with, $m = \frac{GM}{c^2}$ and $c = 1$.

In Figure 2, we have plotted $T_{eff}(r)$ using both Newtonian and Schwarzschild formulae (for two cases : $r_{orb} < R_*$ and $r_{orb} > R_*$), which shows the importance of the general relativistic effect near the surface of the star (most of the x-ray comes from this region).We have taken some arbitrary (but reasonable) values for M, R_* and $\dot{M}$.

The expressions of $T_{eff}(r)$, given before, are for the non-rotating neutron stars. But the neutron stars in LMXBs are expected to be rapidly rotating due to angular momentum transfer by the accreted matter. So we must solve numerically the structure of a neutron star (assuming a particular equation of state, gravitational mass and angular speed) consistently from its centre to infinity (surface-matching should be done) to get its radius, r_{orb} and metric coefficients inside and around it [3]. This will give the corrected temperature profile and hence the spectrum for rapidly rotating neutron stars. It is expected, that at higher x-ray frequencies, the effect of rotation on the spectrum be considerable.

It is also expected, that for the rapidly rotating stars, $\frac{L_S}{L_D} << 1$, which implies that the irradiation of the disk by the surface layer can be neglected in such cases. It justifies our thin disk assumption.

5. Observations

In this section, we will briefly describe some of the observed features of neutron star LMXBs. For details, one may refer [9] and [2].

a) Persistent x-ray emission : Though called 'persistent', luminosity varies to some extent. It is caused by stable disk accretion.

b) Soft X-ray Transients (SXTs) : X-ray luminosity varies from below 10^{33} ergs/sec. to well above 10^{37} ergs/sec. in the range 1 to 10 keV. Its rising time is several days, decay time is several tens of days to more than hundred days, and there may be a gap of several months to tens of years between two such luminosity variations. Disk instability or mass transfer instability may cause it.

c) X-ray burst (type I) : In this case, the rising time is less than 1 sec. to $\sim$10 sec. and the decay time is $\sim$10 sec. to minutes. When the source behaves as a burst-source, the gap between two bursts is 5 minutes to days. The burst activity may remain off for days to months. It is probably caused by the thermo-nuclear flash by the accreted matter on the neutron star surface.

d) X-ray burst (type II) : When the source is active, the gap between two bursts is 7 sec. to 1 hr. and the varying time of luminosity during a particular burst is $\sim$2 sec. to 11 min.. Disk instability may cause such kind of bursts.

e) Quasi Periodic Oscillations (QPOs) : It is the variation of luminosity in the Hz. and kHz. frequency ranges. There are several models, which can explain it to some extent (for example, beat-frequency model [1]).

f) Millisecond (ms) pulsars : They are radio pulsars with pulse (and hence spinning) period in the millisecond range. It is believed that they are old neutron stars, that were spun up by the accretion processes in LMXBs. The considerable overlapping among ms pulsars & binary pulsars and the low observed magnetic field of ms pulsars give observational support to this theoritical idea. The recent (April, 1998) discovery (by RXTE, i.e., Rossi X-ray Timing Explorer) of an ms x-ray pulsar (period 2.49 ms) in an LMXB system (XTE J1808-369) strongly suggests that, LMXBs are the progenitors of at least some of the ms pulsars.

6. Discussions

In this section, we will briefly comment on some relevant points.

a) The temperature profile, mentioned in this paper, will find application in modelling the observed x-ray spectra coming from luminous persistent LMXB sources with neutron star as the central accretor (for example Cygnus x-2).

b) For high Ω_*, surface layer luminosity is expected to be low, hence the irradiation of the disk by the surface layer may be negligible.

c) By calculating rapidly rotating neutron star structure for different equations of state, gravitational masses and angular speeds, one may constrain these, as well as many other quantities, for an observed source.

d) The effect of magnetic field and radiation should be taken into account, in addition to general relativistic effect, in calculating disk structure.

e) With the presently known lifetime of LMXBs, they can not be the progenitors of all the ms pulsars, as the estimated number of ms pulsars in the galaxy is much higher than that of LMXBs.

f) The evolution of angular momentum and many other properties (for example, magnetic field) of an accreting neutron star is still an open question.

References

1. M A Alpar and J Shaham *Nature* **316** 239 (1985)
2. D Bhattacharyya and E P J van den Heuvel *Physics Reports* **203** (1991)
3. G B Cook et al. *Astrophys. J.* **424** 823 (1994)
4. J Frank et al. *Accretion Power in Astrophysics* (Cambridge Univ. Press) 56 (1985)
5. C Jones *Astrophys. J.* **214** 856 (1977)
6. A R King *QJRAS* **29** 1 (1988)
7. R Kippenhahn and A Weigert *Stellar Structure and Evolution* (Heidelberg: Springer-Verlag) 209 (1990)
8. L D Landau and E M Lifshitz *The Classical Theory of Fields* (Pergamon Press) 325 (1971)
9. W H G Lewin et al. (Ed.) *X-ray Binaries* (Cambridge Univ. Press) (1995)
10. A Rai Choudhuri *The Physics of Fluids and Plasmas* (Cambridge Univ. Press) 321 (1998)
11. S Rappaport et al. *Astrophys J.* **275** 713 (1983)
12. S A Rappaport and P C Joss in *Accretion Driven Stellar X-ray Sources*, (Ed.) W H G Lewin and E P J van den Heuvel (Cambridge Univ. Press) 1 (1983)
13. S A Rappaport and P C Joss *Ann. Rev. Astron. Astrophys.* **22** 537 (1984)

14. Lord Rayleigh, *Proc. Roy. Soc. A* **93** 148 (1917)
15. S L Shapiro and S A Teukolsky *Black Holes, White Dwarfs and Neutron Stars* (Wiley-Interscience Publication) 346 (1983)
16. J A van Paradijs in *Timing Neutron Stars* (Ed.) H Ögelman, H. and E P J van den Heuve (Kluwer Academic Publishers, Dordrecht) 191 (1989)
17. F Verbunt and C Zwaan *Astron. Astrophys.* **100** L7 (1981)
18. T Yamada and J Fukue *Publ. Astron. Soc. Japan* **45** 97 (1993)